

Mathematical model for a batch aerated submerged biofilm reactor for organic carbon and nitrogen removal

Youngik Choi[†], Donald Hayes* and Kraig Johnson*

Department of Environmental Engineering at Silla University, San 1-1, Gwaebo-p-dong, Sasang-gu, Busan 617-736, Korea

*Department of Civil and Environmental Engineering at University of Utah, 84112, Salt Lake City, USA

(Received 14 September 2006 • accepted 3 January 2007)

Abstract—An aerated submerged biofilm (ASBF) pilot plant has been developed. The present study optimized an inexpensive method of enhanced wastewater treatment. Over a period of three and one half months, a total of 11 batch runs were performed. By the fourth run, the biofilm had matured to the point that it consumed all the ammonia in 40 hours. The investigation aimed to present mathematical models for describing the dynamic behaviors of the dissolved organic matter removal and nitrification in the Aerated Submerged Biofilm (ASBF) for a batch reactor. Based on the experimental data from the batch system of the ASBF pilot plant, mathematical models were developed to predict dissolved organic matter and ammonia nitrogen removal rates as a function of heterotrophic and autotrophic bacteria populations, dissolved organic matter concentrations, ammonia nitrogen concentrations, dissolved oxygen concentrations, and temperature. The mathematical models for dissolved organic matter and ammonia nitrogen removal in ASBF include two differential equations reflecting heterotrophic and autotrophic bacteria populations, and a number of kinetic parameters. Consequently, the results provide a better insight into the dynamic behaviors of heterotrophic and autotrophic biofilm growth and their practical application to wastewater for dissolved organic matter and ammonia nitrogen removal process.

Key words: Autotrophic Bacteria, Dissolved Oxygen, Heterotrophic Bacteria, Kinetic Parameters, Mathematical Model

INTRODUCTION

The literature review that forms the foundation of this study shows that pH, dissolved oxygen concentrations, and temperature are the most important process parameters for nitrification activity. Researchers report somewhat different values for optimum pH, dissolved oxygen concentrations, and temperature in biological nitrification activity. However, these parameters have not been well researched in the specific case of aerated submerged biofilm reactors.

Even when several aerated submerged biofilm reactors were developed, the nitrification process failed to maximize the surface area, or failed to oxygenate the surface area for biomass. Although increased water and air velocities in and around bio-film led to higher nitrification rates, it was more economical to consider energy consumption and operation of the system at lower air rates. Therefore, advanced new designs to optimize flow rates and energy consumption for aerated submerged bio-film reactors are needed.

Several mathematical models to predict the effluent concentrations for efficient ammonia nitrogen removal have been developed [1-8] but they are either too simple or incomplete, and require too much as-yet-unknown information. None of the models include functions of both temperature and bacteria populations. So far, then, no suitable mathematical model is available for aerated submerged bio-film reactors, and current mathematical modeling cannot provide a suitable solution. However, this paper provides information on the efficiency and mechanisms of dissolved organic matter and ammonia nitrogen removal for both the batch system in the ASBF

pilot plant. This research also examines the mathematical models to predict dissolved organic matter and ammonia nitrogen removal rates as a function of heterotrophic bacteria, autotrophic bacteria, dissolved organic matter concentrations, ammonia nitrogen concentrations, dissolved oxygen concentrations, and temperature. The mathematical models can provide a suitable solution.

MATERIALS AND METHODS

1. Pilot Plant Set-up

The aerated submerged bio-film (ASBF) was tested with a pilot plant installed at the Central Valley Water Reclamation Facility (CVWRF) in the Salt Lake City, USA. CVWRF has a certified water lab. This made possible cross-checking the analysis at the University of Utah lab with the CVWRF lab. In addition, this plant discharges ammonia-laden press supernatant into its own headworks once per week, resulting in high ammonia levels (typically 25 to 30 mg/L) each Wednesday. For batch operation, loading the pilot plant on Wednesdays provided an extra boost of ammonia. The pilot test reactor is located beside an aerated ditch which was located between the primary sedimentation tanks and the trickling filters at CVWRF. This facilitated filling the reactor, as well as emptying it, as the discharge was placed back in the ditch.

The pilot test reactor was located beside an aeration ditch between the primary sedimentation tanks and the trickling filters at CVWRF. This facilitated feeding the reactor in batch mode; reactor effluent was discharged back into the aeration ditch. The pilot plant was constructed from a commercial dumpster with dimensions of 2.4 m by 6.7 m by 0.9 m deep. Inside, 24 ASBF modules were placed so they would be submerged by 0.6 m of primary settled

[†]To whom correspondence should be addressed.

E-mail: coolyoungik@silla.ac.kr

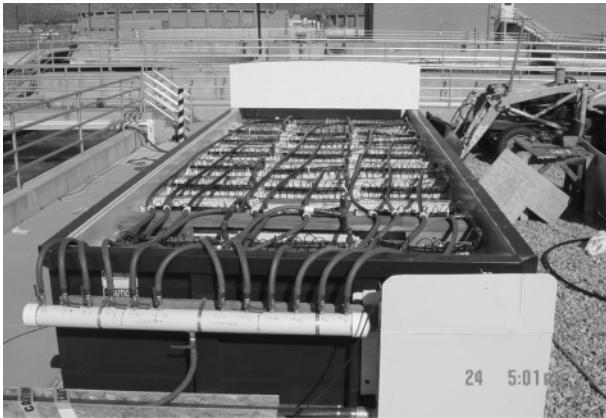


Fig. 1. Top view of pilot plant.

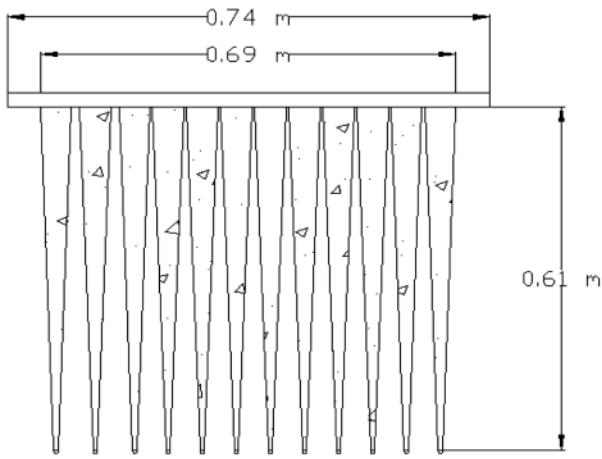


Fig. 2. An end view of twelve wedge shaped concrete panels connected to the top support.

effluent. Each module consisted of 12 panels with a fine bubble distribution tube along the bottom. Bubbles rise up in contact with the surface area on both sides of each panel. Total surface area for bio-film colonization is 302 m². The pilot plant, when filled to the tops of the modules, contained 6,058 liters of wastewater. Total air supply to the modules was 227 liters per minute. Air was supplied with a small shop compressor fitted with regulators and oil/water traps and a knock-out tank. Pressure to the distribution lines was roughly 17.2 kpa. The top of the pilot plant was covered with wooden panels to block the sunlight. Fig. 1 shows a top view of the pilot plant with 24 modules in a plug-flow configuration inside a commercial dumpster. Fig. 2 shows an end view of twelve wedge-shaped concrete panels connected to the top support.

2. Analysis

Field measurements of the wastewater were taken with a Horiba U-10 water checker. These measurements were pH, temperature, dissolved oxygen, turbidity, salinity, and conductivity. Samples were taken at three points in the pilot plant for each sampling event to provide triplicate results for statistical purposes. During a batch run, three sampling events occurred per day. Samples were taken around 1 A.M., 9 A.M. and 5 P.M.. The 1 A.M. samples were taken by using an ISCO auto sampler. All samples were collected and analyzed in

triplicate (n=3) in the experiments. Standard deviations were calculated to assess the reliability of the data.

Measurements of ammonia, nitrate, nitrite, total Kjeldahl nitrogen (TKN) and chemical oxygen demand measured by using Cr (COD_{cr}) were performed at the University of Utah water quality lab. These tests were performed with reagents supplied by Hach and colorimetric analysis on a Hach DR-2500. TKN measurements were performed with a digesdahl digestion method supplied by Hach and colorimetric analysis on a Hach DR-2500. Alkalinity measurements were performed with a titration method.

3. Model Development

The development of mathematical models for the ASBF batch reactor required examination of the fundamental processes and conditions that characterize the system performance. The development of the mathematical model was based on the following assumptions.

- (1) Oxygen diffuses from the gas phase to the bulk liquid phase.
- (2) The BOD/COD ratio is about 0.5.
- (3) Oxygen dissolved organic matter, and ammonia nitrogen diffuse from the bulk liquid to the bio-film.
- (4) Biomass growth is limited by the available oxygen, dissolved organic matter, and ammonia nitrogen; thus, the Monod equation is applicable.
- (5) Dissolved organic matter and ammonia nitrogen are the stoichiometrically limiting nutrients and are completely consumed inside the bio-film.
- (6) The initial heterotrophic and autotrophic bacteria populations are the same at the start of each batch run.
- (7) The heterotrophic and autotrophic bacterial composition of the bio-film changes during each batch run.

4. Batch System

The kinetics of dissolved organic matter and ammonia nitrogen removal and the heterotrophic and autotrophic bacteria populations follow the Michaelis-Menten form. The heterotrophic bacteria populations over time are provided by the differential equation [12, 13]:

$$\frac{dX_H}{dt} = [X_H]_t * \frac{\mu_{max} * [L]_t}{K_s + [L]_t} * \frac{[DO]_t}{K_{O_2} + [DO]_t} - k_d * [X_H]_t \quad (1)$$

The change in BOD with time is provided by the differential equation:

$$\frac{dL}{dt} = - \frac{[X_H]_t * \mu_{max} * [L]_t}{Y} * \frac{[DO]_t}{K_{O_2} + [DO]_t} \quad (2)$$

Eqs. (1) and (2) represent an interrelated set, and they must be solved simultaneously. They cannot be solved analytically but must be solved numerically. These equations, the dissolved oxygen equation, and many kinetic parameters were implemented on MATLAB, which has been designed as a high-performance, numeric computation and visualization software consisting of two independent parts: data input and simulation. There are several ODE solvers, but 4th/5th order Runge-Kutta method is the most popular one. These higher order methods match more terms in Taylor series approximation by taking a weighted average of several derivative approximations. The function of ODE45 is an implementation 4th/5th-order Runge-Kutta method in the MATLAB. MATLAB uses 4th/5th order Runge-Kutta, which also varies the step size h. The value of h is increased where

possible, to speed the calculation and decreased where needed to maintain accuracy. Ordinary Differential Equations (ODEs) are widely used to solve and model physical phenomena. Through the use of conventional mathematical methods such as Runge-Kutta and by using as its default value a very small integration step which is 0.1, MATLAB can easily solve nonlinear differential equation systems.

The dissolved oxygen concentration can be expressed by this equation [14]:

$$DO = S - D \quad (3)$$

With the assumption that the dissolved oxygen saturation concentration is constant, the differential of Eq. (3) provides Eq. (4).

$$\frac{dDO}{dt} = - \frac{dD}{dt} \quad (4)$$

$$\frac{dD}{dt} = - D_s * \exp(-kla_T * t) \quad (5)$$

Inserting Eq. (5) [14] into Eq. (4) yields the dissolved oxygen concentration change over time is provided by the differential equation:

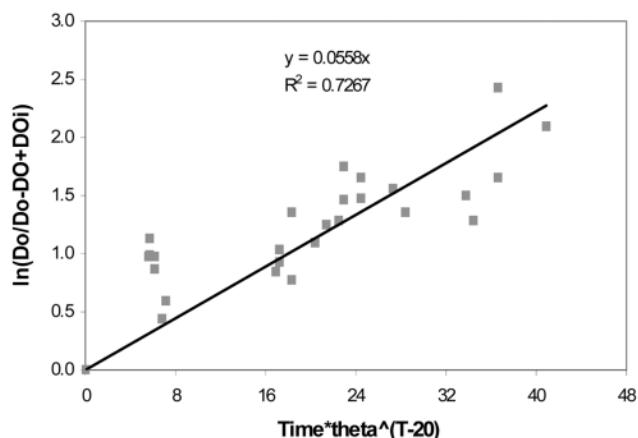


Fig. 3. Overall gas transfer coefficient at 20 °C.

Table 1. Dissolved oxygen data in all batch reactor

Batch #	Temp. (°C)	Time (hours)	$\ln(Do/Do-DOi)$	DO (mg/L)	DO (mg/L)	Time* $^\wedge(T-20\text{ °C})$
Batch 4	15	0	0.00	0.22	6.28	0.00
		8	0.59	3.02		7.11
		24	1.25	4.70		21.32
		32	1.36	4.89		28.42
Batch 5	13.2	0	0.00	0.38	6.48	0.00
		8	0.44	2.69		6.81
		24	1.10	4.70		20.43
		32	1.55	5.49		27.23
		48	2.09	6.06		40.85
Batch 6	8.6	0	0.00	0.51	7.27	0.00
		8	0.87	4.72		6.10
		24	1.35	5.90		18.31
		32	1.48	6.12		24.42
		48	2.43	7.14		36.63
Batch 7	8.6	0	0.00	0.58	7.20	0.00
		8	0.98	5.08		6.10
		24	0.77	4.45		18.31
		32	1.65	6.40		24.42
		48	1.65	6.40		36.63
Batch 8	6	0	0.00	0.55	7.75	0.00
		8	0.99	5.41		5.74
		24	0.93	5.23		17.22
		32	1.47	6.51		22.96
		48	1.29	6.17		34.44
Batch 10	6	0	0.00	0.46	7.84	0.00
		8	1.13	5.78		5.74
		24	1.04	5.53		17.22
		32	1.75	6.94		22.96
Batch 11	5.1	0	0.00	0.41	8.07	0.00
		8	0.98	5.45		5.62
		24	0.84	5.01		16.86
		32	1.28	6.24		22.47
		48	1.50	6.68		33.71

$$\frac{dDO}{dt} = D_o * \exp(-kla_T * t) \quad (6)$$

Temperature effects on the coefficients were also considered because of the wide range of wastewater temperatures during the batch tests. All kinetic parameters were adjusted for temperature by using an Arrhenius relationship: $k_T = k_{20} \theta^{T-20}$ [12-14].

$$kla_T = (kla)_{20} \theta^{T-20} \quad (7)$$

Plotting $t * \theta^{T-20}$ versus $\ln(D_o/D_o + DO_{initial} - DO)$ provides a slope as $(kla)_{20}$. Fig. 3 shows $t * \theta^{T-20}$ vs $\ln(D_o/D_o + DO_{initial} - DO)$. The slope represents $(kla)_{20}$.

The overall gas transfer coefficient at 20 °C is 0.0558 hour⁻¹. The overall gas transfer coefficient *kla* is also function of temperature, intensity of mixing, size of bubbles, volume of water and constituents in the water. The higher *kla* means that the diffuser is more effective in driving oxygen into the water, apparently the more economical to operate in the wastewater treatment plants. In the systems used in the field of wastewater treatment, the change in concentration of the gas with time is generally proportional to the difference between the existing concentration at time *t* and the saturation concentration of the gas in solution. It can be expressed as $S = -0.2 * \text{temperature} + 9.5$. Dissolved oxygen data in all batch reactors are shown in Table 1. The change in autotrophic bacteria populations through time is provided by this differential equation [12,13]:

$$\frac{dX_A}{dt} = [X_A]_t * \mu_{max} * \frac{[NH_4^+]_t * [DO]_t}{K_s + [NH_4^+]_t} - k_d * [X_A]_t \quad (8)$$

The rate of ammonia nitrogen concentration change with time is provided by the differential equation:

Table 2. Kinetic parameters of heterotrophic bacteria^a for the literature range (T=20 °C)

Heterotrophs	Literature range	Reference
μ_{max} (day ⁻¹)	4.0	[9]
	3.8	[10]
	2.1	[11]
	5.0	[12]
	6.0	[13]
k_s (mg/L)	10	[9]
	30	[11]
	60	[12]
	20	[13]
Y (mg/mg)	0.57	[9]
	0.50	[10]
	0.63	[11]
	0.60	[12]
	0.67	[13]
k_d (day ⁻¹)	0.15	[9]
	0.12	[11]
	0.62	[13]
k_{O_2} (mg/L)	0.25	[10]

^aFor the purposes of this research, bacteria that obtain their energy from the oxidation of organic compounds are called heterotrophs generically, even though some heterotrophs may also obtain energy from inorganic compounds.

$$\frac{dNH_4^+}{dt} = - \frac{[X_A]_t * \mu_{max} * [NH_4^+]_t * [DO]_t}{Y * K_s + [NH_4^+]_t} \frac{[DO]_t}{K_{O_2} + [DO]_t} \quad (9)$$

Eqs. (8) and (9) also represent an integrated relationship which must be solved together numerically rather than analytically. These equations, the dissolved oxygen equation, and many kinetic parameters were implemented on MATLAB software. These equations and many kinetic parameters were implemented on MATLAB. MATLAB codes were used for this mathematical modeling.

5. Determination of Parameters

The kinetic coefficients adapted for wastewater are provided in Table 2 and Table 3, which give a summary of the parameters found from kinetic parameters of heterotrophic and autotrophic bacteria suggested in the literature range at temperature 20 °C respectively. The series of differential equations above govern the dissolved organic matter and ammonia nitrogen removal from the wastewater in the ASBF batch reactor. A model can be used effectively only if the parameters are known. The values of these parameters can be determined through a mathematical calibration procedure that requires suitable experimental data. However, there is not complete set of experimental data for these parameters in the ASBF system sufficient to calibrate the model.

The parameters in the heterotrophic bacteria growth rates and the dissolved organic matter removal rates, except the bacteria decay constant, were also selected from the literature data at its mid-point. Even though the mean value of the bacteria decay constant in those

Table 3. Kinetic parameters of autotrophic bacteria^a for the literature range (T=20 °C)

Heterotrophs	Literature range	Reference
μ_{max} (day ⁻¹)	0.83	[9]
	0.58	[10]
	0.86	[14]
	0.21	[11]
	1.0	[12]
	0.80	[13]
k_s (mg/L)	2.8	[9]
	0.8	[10]
	1.0	[11]
	1.4	[12]
	1.0	[13]
Y (mg/mg)	0.24	[9]
	0.24	[11]
	0.20	[12]
	0.24	[13]
k_d (day ⁻¹)	0.05	[9]
	0.05	[11]
	0.05	[12]
k_{O_2} (mg/L)	1.0(15 °C)	[1]
	0.3	[9]
	0.5	[10]
	0.4	[13]

^aFor the purposes of this research, nitrifying bacteria that derive their energy from the oxidation of inorganic compounds are referred to generically autotrophs.

circumstances is 0.37 day⁻¹, the value of the bacteria decay constant that was selected was 2.16 day⁻¹. The reason for this selection came from the following observation: the experimental data of all batch reactors showed that dissolved organic matter concentrations were removed rapidly during the first 16 hours, and that the dissolved organic matter remained about 20-30 mg/L up to 48 hours. This meant that the heterotrophic bacteria decay constant would be higher than the constant indicated in the literature range.

All of the parameters relative to the autotrophic bacteria growth rates and the ammonia nitrogen removal rates were selected from data provided by the literature. Since the range of the parameters varies due to temperature, mean value of the temperature range was selected; this was determined to be 20 °C. The effects of temperature on the maximum growth rates and the half-velocity constant were also considered because of the wide range of wastewater temperatures during the batch tests. All kinetic parameters were adjusted for temperature with the formulations: $\mu_{maxT} = m_{max20C} * \theta^{T-20C}$ and $K_{s,T} = K_{s,20C} * \theta^{T-20C}$. For the temperature activity coefficient θ , a typical value is 1.04 for biological processes [14]. The kinetic parameters used in the MATLAB model for the heterotrophic and autotrophic bacteria growth rates are presented in Table 4 and Table 5.

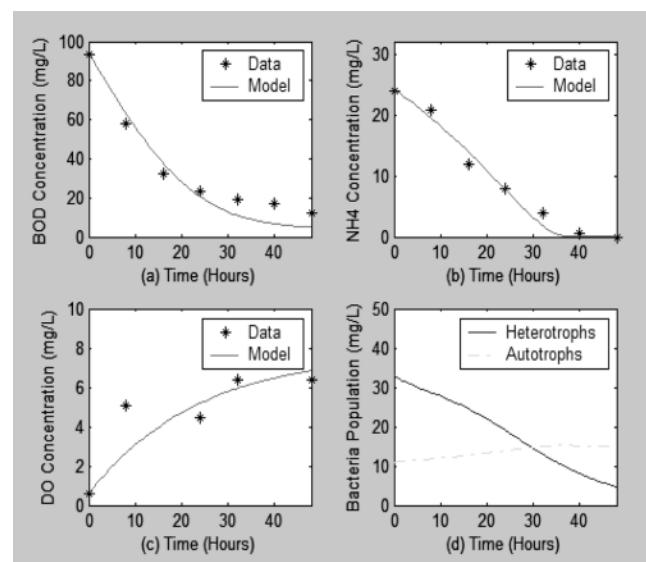
6. Analysis of Mathematical Models

Data were collected over a series of 11 batch runs, as the ambient air and water temperature dropped. Significant among the results were rapid rates of nitrification at temperatures down to 3.3 °C in the batch mode. Batch runs #1, #2 and #3 were start-up of ASBF reactor. Eight batch runs were conducted from the first part of October to the middle of December. The ammonia nitrogen removal rate of Batch run #9, initiated on December 2. During this run, no air was turned on to the bio-film in an effort to assess the natural oxygenation potential through the surface of the water.

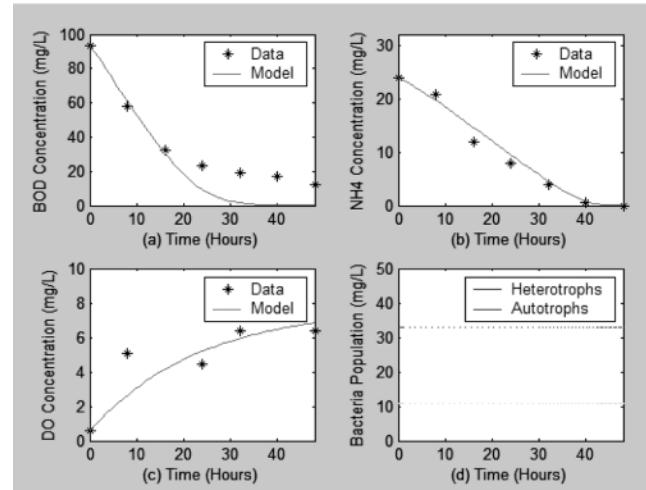
To develop batch mathematical models, the Monod equation was applied. The models consist of maximum growth rates (μ_{max}), half-velocity constants (k_s), dissolved oxygen concentrations (DO), dissolved oxygen half-velocity constants (k_{O_2}), yield coefficients (Y), bacteria decay constants (k_d), water temperatures (T), and concen-

trations of substrates (L and NH_4^+) and heterotrophic and autotrophic bacteria populations (X_H and X_A). This section describes the numeric implementation of these equations for dissolved organic matter and ammonia nitrogen removal in the ASBF. Model fits are illustrated in Fig. 4(A).

In order to verify an advantage of the proposed model, a comparison with another model which is fixed concentrations of heterotrophic and autotrophic bacteria through the experiment (batch run #7) is tested. Our mathematical models are proposed to describe the dynamic behavior of the dissolved organic matter removal and nitrification in the ASBF with heterotrophic and autotrophic bacteria population dynamics with great confidence (see Fig. 4). How-



(A)



(B)

Fig. 4. Plots for (a) measured, simulated BOD removal, (b) measured, simulated ammonia nitrogen removal, (c) measured, simulated DO and (d) simulated bacteria populations for Batch #7 in the (A) and plots for (a) measured, simulated BOD removal, (b) measured, simulated ammonia nitrogen removal, (c) measured, simulated DO and (d) simulated bacteria populations for Batch #7 with fixed concentrations with bacteria populations in the (B).

Table 4. The kinetic parameters used for the heterotrophic bacteria growth rates

Parameters	Selected value	Literature range
μ_{max} (day ⁻¹)	$0.169 * (1.04)^{(temp-20\text{ }^{\circ}\text{C})}$	0.0875-0.25
k_s (mg/L)	$33 * (1.04)^{(temp-20\text{ }^{\circ}\text{C})}$	10-60
Y (mg/mg)	0.59	0.50-0.67
k_d (day ⁻¹)	0.09	0.005-0.026
k_{O_2} (mg/L)	0.25	0.25

Table 5. The kinetic parameters used for the autotrophic bacteria growth rates

Parameters	Selected value	Literature range
μ_{max} (day ⁻¹)	$0.025 * (1.04)^{(temp-20\text{ }^{\circ}\text{C})}$	0.0875-0.0417
k_s (mg/L)	$1.8 * (1.04)^{(temp-20\text{ }^{\circ}\text{C})}$	0.8-2.8
Y (mg/mg)	0.22	0.20-0.24
k_d (day ⁻¹)	0.0021	0.0021
k_{O_2} (mg/L)	0.65	0.3-1

ever the previous mathematical models do not provide the removal of dissolved organic matter and ammonia nitrogen with heterotrophic and autotrophic bacteria population dynamics (see Fig. 4(B)).

The pilot plant results matched and verified the mathematical model results. The successful application of a model requires calibration that includes comparisons of model predicted results with measured data produced during the experiments. Model default values were used to show rational simulations of the performance of pilot scale for dissolved organic matter and ammonia nitrogen removal rates. There is little doubt that more precise simulations could be achieved if kinetic coefficients were tested and measured. Successful model application requires a comparison of the results predicted by the model with measured data provided by the experiments. By performing simulations, the flow rates and dissolved oxygen concentrations will be controlled. Consequently, the results presented in this paper provide a better insight into the dynamics of heterotrophic and autotrophic bio-film growth and their practical application to wastewater dissolved organic matter and ammonia nitrogen removal process. This preliminary work can save time, manpower, and the cost of chemicals in the future.

RESULTS AND DISCUSSION

1. Model Verification

The model verification in this research was based on a comparison of the experimental results, derived from data from seven experimental batch sets in the plant pilot, with the model predicted values. Positive verification of the model occurred when agreement between the experimental results of the seven experimental batch runs and the predicted values was obtained through use of the same set of coefficients. Because the use of the same parameters was justified, it became clear that the model could successfully predict the pilot performance for dissolved organic matter and ammonia nitrogen concentrations. There was noteworthy agreement between the experimental data and the predicted data: the maximum difference for dissolved organic matter removal between the observed and predicted values is 20 mg/L in Batch Run #4 the maximum difference for ammonia nitrogen removal between the observed and predicted values is 3.2 mg/L in Batch Run #4.

Wastewater with higher ratio produces higher heterotrophic bacteria populations and lower autotrophic bacteria populations. BOD/ $\text{NH}_4^+ - \text{N}$ ratios are from 1.9 to 57.5 in the batch system. As BOD/ $\text{NH}_4^+ - \text{N}$ ratios are increased, ammonia nitrogen removal rates have a tendency to be decreased. This has been confirmed that a high BOD/ $\text{NH}_4^+ - \text{N}$ ratio is directly related to the growth of heterotrophic bacteria. After BOD/ $\text{NH}_4^+ - \text{N}$ ratio reached around 2, nitrification activity further improved. At this time, nitrification rate was as high as 0.9 mg/L·hour. This indicates that nitrifying bacteria existing in the biofilm would gradually acclimatize and multiply in the wastewater. The microbial community structure of the biofilm changed through gradual decrease in BOD/ $\text{NH}_4^+ - \text{N}$ ratio. As the fraction of heterotrophic bacteria in the biofilm gradually decreased, the population of ammonia-oxidizing bacteria increased their number and extended their existence area to the outer part of the biofilm in place of heterotrophs.

It is interesting to evaluate the model ammonia removal rate as a function of the BOD/ $\text{NH}_4^+ - \text{N}$ ratio. Fig. 5 shows the model am-

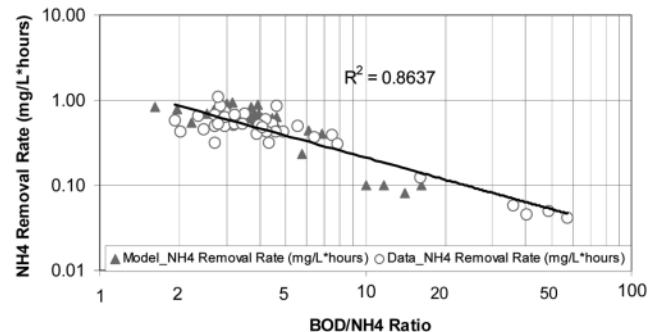


Fig. 5. Model ammonia nitrogen removal rate vs. BOD/ $\text{NH}_4^+ - \text{N}$ ratio.

monia removal rate as a function of the BOD/ $\text{NH}_4^+ - \text{N}$ plotted in log-log scale. This model has been verified that as BOD/ $\text{NH}_4^+ - \text{N}$ ratios are increased, ammonia nitrogen removal rates also have a tendency to be decreased.

At high concentration of ammonia nitrogen, the system is a zero-order or half-order with respect to ammonia nitrogen. And the kinetic order can be first-order for ammonia nitrogen at low concentrations of ammonia nitrogen. The data qualitatively indicate the tendency that there is a gradual transition from a zero-order reaction at higher concentrations of ammonia nitrogen to half-order to first order reaction at lower concentrations of ammonia nitrogen. The models have also represented that at high ammonia nitrogen concentrations, a region is zero-order for ammonia nitrogen. At somewhat lower concentrations, the system is half order for ammonia nitrogen and at lower concentrations, the system becomes first-order for ammonia nitrogen. Fig. 6 shows the model nitrification rates as a function of the model ammonia nitrogen concentrations plotted in log-log scale. It could be also concluded that the model ammonia nitrogen removal rate is first-order at particularly low concentrations of ammonia nitrogen, the rate is half-order around 5-10 mg/L of ammonia nitrogen, and the rate is zero-order at high concentrations (above 11 mg/L).

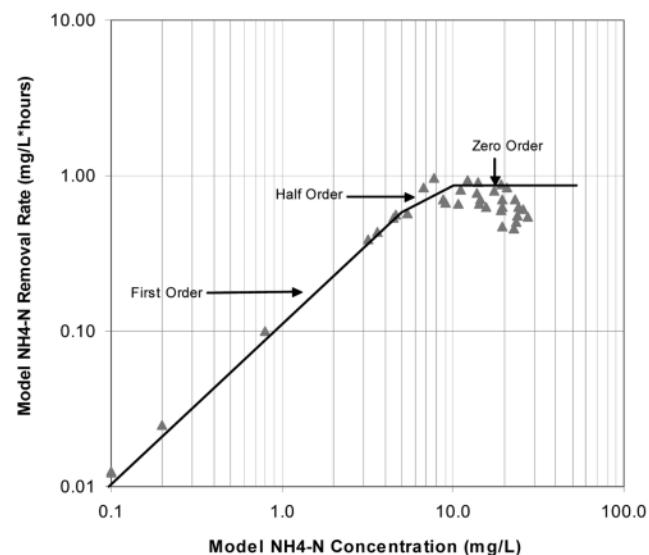


Fig. 6. Model nitrification rate vs. model ammonia nitrogen concentration.

Table 6. Standard error of estimate of BOD in batch system

Time (hours)	Batch						
	#4	#5	#6	#7	#8	#10	#11
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	400.0	198.8	50.4	30.3	412.1	53.3	134.6
16	33.6	51.8	108.2	29.2	380.3	34.8	151.3
24	106.1	278.9	204.5	9.0	77.4	13.7	96.0
32	313.3	292.4	259.2	64.0	0.5	41.0	53.3
40	372.5	249.6	234.1	106.1	20.3	44.9	34.8
48	349.7	182.3	207.4	51.8	28.1	0.0	30.3
S _e	15.0	13.4	12.3	6.4	11.5	5.6	8.5

Table 7. Standard error of estimate of ammonia nitrogen in batch system

Time (hours)	Batch						
	#4	#5	#6	#7	#8	#10	#11
0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	3.6	0.4	0.0	1.4	0.4	0.2	1.2
16	0.6	2.0	1.0	3.6	0.3	1.7	2.9
24	10.2	10.9	0.0	0.0	0.8	8.4	5.3
32	9.0	13.0	2.6	4.4	15.2	2.9	18.5
40	0.0	0.2	1.2	0.3	8.4	0.0	17.6
48	0.0	0.0	0.1	0.0	0.0	0.0	0.0
S _e	1.8	1.9	0.8	1.2	2.1	1.5	2.6

The standard error of the estimate is a measure of the accuracy of predictions made with regression line. The regression line seeks to minimize the sum of the squared errors of prediction. The standard error of estimate is calculated by Eq. (10) [16].

$$S_e = \left(\frac{1}{n} \sum_{i=1}^n [y_i - \hat{y}_i]^2 \right)^{1/2} \quad (10)$$

The standard errors of estimate of dissolved organic matter and ammonia nitrogen removal are shown in Table 6 and Table 7. The smallest standard error of estimate of dissolved organic matter and ammonia nitrogen remaining in all batch systems is 5.6 in batch #10 and 0.8 observed in batch #6, respectively. This agreement between the experimental values and the predicated values positively verified the mathematical models.

It is clear from this research that the mathematical models accurately predicted the pilot performance for dissolved organic matter and ammonia nitrogen concentrations, since noteworthy agreement occurred between the experimental data and the predicted data.

CONCLUSIONS

The results of this mathematical modeling provide useful information on the factors that significantly affect the ASBF system performance. They also provide a range of design parameters for municipal wastewater treatment. No model so far has been able to predict satisfactorily the removal of dissolved organic matter and ammonia nitrogen with heterotrophic and autotrophic bacteria population dynamics with great confidence. The results of this study yield the following specific conclusions:

1. Mathematical models which describe the kinetics of the heterotrophic bacteria, autotrophic bacteria, dissolved oxygen concentrations, and dissolved organic matter and ammonia nitrogen removal were formulated and implemented in MATLAB software numerically.

2. The kinetic parameters are selected from a mean value from literature data that was determined at 20 °C.

3. The model can be used to explain and make acceptable prediction on the performance of the ASBF process.

4. The mathematical model successfully explained the dynamic behavior of dissolved organic matter removal and nitrification.

5. Further development of this model will be made possible by additional experiments that allow identification of the heterotrophic and autotrophic bacteria populations over the biofilm.

NOMENCLATURE

A : a constant [mg/l]
C : concentration [mg/l]
D : dissolved oxygen deficit [mg/l]
D_o : initial dissolved oxygen deficit [mg/l]
DO : dissolved oxygen concentration [mg/l]
E : turbulent diffusion [m²/hour]
k : rate constant [hour⁻¹]
k_{d,A} : autotrophic bacteria decay constant [hour⁻¹]
k_{d,H} : heterotrophic bacteria decay constant [hour⁻¹]
k_{l_aT} : overall gas transfer coefficient at temperature [hour⁻¹]
(k_{l_a})₂₀ : overall gas transfer coefficient at 20 °C [hour⁻¹]
k_{l_{o₂A}} : dissolved oxygen half-velocity constant for autotrophic bacteria [mg/l]
k_{l_{o₂H}} : dissolved oxygen half-velocity constant for heterotrophic bacteria [mg/l]
k_{s,A} : autotrophic bacteria half-velocity constant [mg/l]
k_{s,H} : heterotrophic bacteria half-velocity constant [mg/l]
L : biochemical oxygen demand (BOD) [mg/l]
n : number of independent observations
NH₄⁺ : ammonia nitrogen concentration [mg/l]
S : dissolved oxygen saturation concentration [mg/l]
S_e : standard error of estimate
t : time of reaction [hour⁻¹]
X_A : autotrophic bacteria populations [mg/l]
X_H : heterotrophic bacteria populations [mg/l]
Y_A : autotrophic bacteria yield [mg/l]
Y_H : heterotrophic bacteria yield [mg/l]
y_i : observed values of the dependent variables
ŷ_i : predicted values of the dependent variables

Greek Letters

θ : temperature activity coefficient, typical value is 1.024 for oxygen
μ_{max,A} : autotrophic bacteria maximum growth rate [hour⁻¹]
μ_{max,H} : heterotrophic bacteria maximum growth rate [hour⁻¹]

REFERENCES

1. D. Barnes and P. J. Bliss, *Biological control of nitrogen in wastewater treatment*, E. & F. N. Spon Ltd. (1983).

2. D. V. Vayenas and G. Lyberatos, *Wat. Res.*, **28**, 1275 (1994).
3. Y. Argaman, G. Papkov, A. Ostfeld and D. Rubin, *J. Envir. Engr.*, **125**, 608 (1999).
4. R. S. Bernardes, H. Spanjers and A. Klapwijk, *Bioresource Technology*, **67**, 177 (1999).
5. G. Cao, Q. Zhao, X. Sun and T. Zhang, *Enzyme and Microbial Technology*, **30**, 49 (2002).
6. D. Sen, C. W. Randall, R. R. Copithorn and T. J. Grizzard, *Wat. Sci. Tech.*, **26**, 2237 (1992).
7. G. G. Bortone and A. Tilche, *Wat. Sci. Tech.*, **35**, 113 (1997).
8. S. C. Reed, *Natural systems for waste management and treatment*, McGraw-Hill., New York (1995).
9. M. Fruhen, E. Christian, W. Gujer and O. Wanner, *Wat. Sci. Tech.*, **23**, 1365 (1991).
10. K. Hoen, M. Schuhene and M. Kohne, *Wat. Sci. Tech.*, **33**, 223 (1996).
11. S. Murat, G. Insel, N. Artan and D. Orhon, *Wat. Sci. Tech.*, **48**, 319 (2003).
12. G. Tchobanoglous and F. L. Burton, *Wastewater engineering*, McGraw-Hill., New York (1991).
13. U.S. Environmental Protection Agency, *Nitrogen control*, Washington, D.C. (1993).
14. P. Harremoes, A. Haarbo, M. Winther-Nielsen and C. Thirsing, *Wat. Sci. Tech.*, **38**, 219 (1998).
15. C. S. Chapra, *Surface water-quality modeling*, McGraw-Hill., New York (1997).
16. A. E. McBean and A. F. Rovers, *Statistical procedures for analysis of environmental monitoring data & risk assessment*, Prentice Hall PTR Environmental Management & Engineering Series, Volume 3 (1998).